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Costly Learning vs. Risk Taking in Financial Contracts: An Application to the International Capital Market

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*Information frictions can lead to sudden cut off of international capital flows to emerging market economies. These situations of financial distress have become known as sudden stops. These are typically treated as exogenous events with sharp falls in output and employment. This paper develops a theory of sudden stops which explains the existence and nature of these events without imposing exogenous restrictions. We build a model that investigates the trade-off between costly-learning and risk-taking in international lending contracts that lead to sudden stops. To learn relevant information about the borrower and the economic environment, the international lender stages the capital to the borrower. This learning process, however, comes at the risk of losing the funds provided in earlier stages. Thus, in designing the contract, the lender considers this trade-off by optimally dividing the loan between short-term and long-term. Our model generates **a** debt maturity structure that matches the documented fact that the share of short-term debt in the overall debt is higher than long-run debt in emerging markets. One key policy goal to deal with informational frictions is to improve information channels. Recent efforts by the IMF and developing countries to improve data dissemination move in the direction of this goal*.

JEL Classification: D82, F34, F37, G15.

1.Introduction

Emerging markets can find themselves suddenly cut off from international capital markets. For example, in 1994, Mexico experienced a reduction in net private capital flows equivalent to almost 4 percent of its GDP followed by a further drop of more than 5 percent in 1995. Output dropped 6 percent in the crisis year, and the economy was plunged into a systemic banking crisis until 1997. Korea, Indonesia, and other emerging Asian markets, faced a similar situation in late 1997, when they were unable to obtain desperately needed private foreign financing. These episodes of financial distress have become known as *sudden stops*.[i] These are typically treated as exogenous events with sharp falls in output and employment. This paper develops a theory of sudden stops which explains the existence and nature of financial crises without imposing exogenous restrictions.

Sudden stops are seriously at odd with the majority of existing models of current account determination, which rely on perfect capital markets.[ii] This failure of the existing theory has led to extensive research seeking to build models that can deliver predictions consistent with the sudden stop phenomenon.[iii] The present study proposes to complement the literature by introducing a trade-off between costly learning and risk-taking, and examines its impact on international capital market decisions and outcomes. The framwork prosed hereenriches the description of the financial crisis in a

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way that makes it considerably easier to motivate endogenous sudden stops. The model is in line with the credit rationing literature that has come to the view that the termination of financial contracts may actually be an equilibrium phenomenon driven by the asymmetry of information between borrowers and lenders.

One major result of the present study is that sudden stops can be rationalized as part of an optimal lending strategy in the face of asymmetric information. This result is consistent with previous empirical findings (see for example, Rodrik and Velasco ,1999). Moreover, the model generates a share of long-term debt smaller than the share of short-term debt in the overall loan. This debt maturity structure matches the documented fact that the share of short-term debt in the overall debt is higher in emerging markets.

The remainder of the paper is organized as follows. Section 2 survys the literature and compare the findings of this study with the results of existant literature. Section 3 describes the economic environment. Section 4 defines potential candidate equilibria and discusses conditions necessary for their existence. Section 5 characterizes the optimal contract. Section 6 discusses the results. Finally, concluding remarks are given in section 7. Apendix section provides proof.

2. Literature Review

The framework developed in this paper explores three key features of the 1990s emerging economies financial crises. First, many observers pointed to the growing ratios of short-term liabilities relative to long-term liabilities as a fundamental source of financial distress.[iv]We argue that, while there is theoretical and empirical evidence supporting this view, even long-term loans will not prevent the crisis from occurring in the presence of asymmetric information, and this finding contradicts the traditional view of a large part of the literature.[v]

Second, as Pomerleano (1999) points out, the high profitability of investment in the East Asian economies[vi] led to financial excesses that violated prudent financial practices, and eventually led to inevitable financial distress. In fact, the Average Return on Equity Performance during 1992-96 in these economies was outstanding, reaching 12.7 percent. In addition, projected growth (by IMF) has reached of 6 percent per year for developing countries as compared to 2.5 percent for industrialized nations.These facts offer a plausible explanation why international investors target emerging market economies even though a lack of market discipline and asymmetric information characterize the economic environment in these markets.

Finally, because information acquisition is costly and “depreciates” quickly, international lenders have less incentive to pay for fixed country-specific information. Calvo and Mendoza (2000) show that, in the presence of information frictions, financial globalization may have *reduced* the incentive to pay fixed country-specific information costs. As a result, the expected-utility gain of paying information costs declines. Their calibrated model show that as the number of emerging markets rises from 2 to 20, the utility gain of paying for country-specific information falls from 60 percent to 10 percent in terms of the mean portfolio return. Hence, international lenders might not be willing to pay the costs needed to verify the state of the world, and the equilibrium outcome would

be the same as the equilibrium outcome of an environment characterized by information frictions (like those in this model).

My work builds on contributions by Bolton and Scharfstein (1990) and Stiglitz and Weiss (1983). Both studies have argued that the termination threat is an effective incentive device. Their analyses, however, differ from mine in three ways: (1) The contracts they investigate do not consider learning; and the first-period capital they consider is a exogenous constant amount while it is an endogenous continuous amount of resources in my model. (2) The contracts that Stiglitz and Weiss (1983) consider are not optimal while the contract I have designed is. Bolton and Scharfstein (1990) consider an optimal contract, but their incentive problem concerns observability of profit while returns are instead observable in my model. (3) Apart from the focus on termination threats as an incentive compatible device in credit and labour market contracts, none of these studies investigate similar issues in the international capital markets context. My approach here is an application of this framework to an international capital market.

3.The Environment

Consider a small country inhabited by a single representative agent, domestic entrepreneur or borrower. The domestic entrepreneur, has a project that lasts for two periods (three-date model: $t=0$, $t=1$, and $t=2$) and offers stochastic returns. The project requires K units in capital and produces output only on date 2. The domestic entrepreneur starts with zero wealth[vii] and thus requires a lender to finance her investment. There is only one foreign bank that can finance the project. Both the borrower and the lender are risk neutral (so both are uninterested in consumption smoothing), and the utility for both depends on last-period consumption, which imply that everyone acts to maximize expected final payoff. Both borrower and lender discount flows using the same subjective discount rate, $\beta > 0$.

Although K is fixed, the bank can divide the required amount into two installments over the two periods; the first installment represents a long-term loan, and the second installment represents a short-term loan. So, the project could be financed with long-term debt or a mix of short-term and long-term debt. If it consists of long-term financing, then the bank provides K at $t=0$ and collects repayment at $t=2$. If financing includes both short-term and long-term debt, then the long-term debt consists of the first installment, a fraction, σK, where $0 \le \sigma \le 1$, of the required capital provided at $t=0$ and to be paid back at $t=2$. The short-term debt consists of the remaining fraction of required capital input, $(1-\sigma)K$ provided at $t=1$ and to be paid back at $t=2$.[viii] Note here that an increase of long-term debt (i.e., σ high) results in a decrease of short-term debt (i.e., $(1-\sigma)$ drops) and vice versa. In other words, the relative size of short- versus long-term debt is determined by the value of σ.

If undertaken and *completed*, the project yields verifiable income $y > 0$. If an amount $\tilde{K} < K$ is invested over the two periods, then $y = 0$. To keep the analysis tractable, we consider that the production function is linear in capital, that is

$$y = AK, \tag{1}$$

where $A > 1$ is a productivity parameter. Furthermore, we assume that there is no uncertainty in the second period; that is, if the entrepreneur invests in period 2, she will be able to generate output and repay the lender.

In the first period, once the loan is secured (i.e., after first period transfer, σK, has been made), nature assigns an alternative investment opportunity to the borrower. After observing this opportunity, the entrepreneur might use (i.e., divert) all the provided funds to the new investment opportunity; that is, she may misbehave. The alternative investment opportunity consists of a constant returns-to-scale investment technology that returns R per unit invested per period. $R \in \{0, \overline{R}\}$ is a random return on the alternative investment opportunity, drawn by nature and *observed by the borrower only*. If the entrepreneur diverts the funds granted to her, the bank cannot expect any repayment. It is also assumed that $\overline{R} > A$. So, because of the risk neutrality assumption, the entrepreneur uses the whole amount received in the alternative investment opportunity if the choice of nature is $\overline{R}$. Thus, misbehaving results in zero output, $y = 0$, with probability one and a private benefit of $\overline{R}$ per unit invested. Alternatively, the entrepreneur invests the borrowed funds in the project if $R = 0$.

In addition to the adverse selection problem, first-period investment, if any, is subject to an exogenous shock, $\varepsilon \in \{0,1\}$. The prior probability that $\varepsilon = 0$ is μ; that is, $Pr(\varepsilon = 0) = \mu$.[ix] Thus, if K units of capital are invested over the two periods, returns of the project are

$$y = \begin{cases} 0 & with\ probability\ \mu \\ AK & with\ probability\ 1-\mu \end{cases}$$

This means that investing σK in the first period does not necessarily result in a good outcome. Let $I = \varepsilon\sigma K$ denote first period observed investment. If ε equals 0, first period investment, I, would be 0, too, and the domestic entrepreneur needs to borrow K units of capital in the second period for the project to be successful. Behaving in the second period (i.e., in first period $R = 0$) yields a positive output $y = AK$ with probability one and no return from the alternative investment opportunity to the domestic entrepreneur.

Throughout the paper, a "good" borrower, or an (R=0)-type, refers to an entrepreneur who is assigned R=0 by nature, while a "bad" borrower, or an ($R = \overline{R}$)-type, is an entrepreneur who is assigned $R = \overline{R}$ by nature. The lender knows only the distribution of R. Let ρ_0 denote the initial prior that the borrower is of the good type; that is, $Pr(R = 0) = \rho_0$ (i.e., the prior about the borrower being a bad type is $1 - \rho_0$). After realization of the random variables (ε and R), and conditional on the outcome of events, the lender updates her prior with the newly arrived information (observed investment, I). Therefore, updating of beliefs, ρ_1, is the result of a mapping from the public information to the unit interval. This mapping, or learning process, is modeled as Bayesian updating based on the public information available to the lender. Thus, learning proceeds as follows:

$$\rho_1 = Pr(R=0 \mid \rho_0, I) = \frac{Pr(\rho_0, I \mid R=0)}{Pr(\rho_0, I)} Pr(R=0). \quad (2)$$

where $Pr(R=0 \mid \rho_0, I)$ is probability that $R=0$ conditional on observing $I=0$, and given the prior ρ_0. Two kinds of inefficiencies might take place. The first one refers to a situation in which investment has been undertaken; that is, the borrower did invest σK, but a "bad" shock, $\varepsilon = 0$, hit the investment. The result is then actual (or observed) investment of zero ($I=0$), and the bank may stop funding the borrower when she should not. The second inefficiency arises when the bank decides to continue financing the project on the basis of a "wrong" belief that the borrower is ($R=0$)-type. As a result, additional funding will be provided to the borrower, who is going to use it in the alternative investment opportunity and not in the project. This scenario means that the bank loses its money and "rewards" the entrepreneur when it should not.[x]

Note that the non-observability of the investment decision is a standard scenario. Projects often require specific human capital or the design of blueprints for machinery, buildings or logistics. Or an entrepreneur may spend a great deal of time reading and designing. The bank is unlikely to observe whether the efforts are directed towards the project or whether blueprints are competently drafted.

3.1 Contracting

At $t=0$ the foreign bank makes a take-it-or-leave-it offer to the entrepreneur. The offer consists of a contract with the following terms: cash flows from the lender to the borrower at $t=0$ and $t=1$, suspension policies, and repayment schedule at $t=2$. These terms can be contingent on all information available. Note that the entrepreneur accepts the offer if the contract provides non-negative expected value.

A lending contract specifies an initial loan size, σ, (a fraction of K) at $t=0$; a repayment schedule (at $t=2$) conditional on I:

$$repayment = \begin{cases} Z & if\ I > 0 \\ Z_0 & if\ I = 0; \end{cases} \quad (3)$$

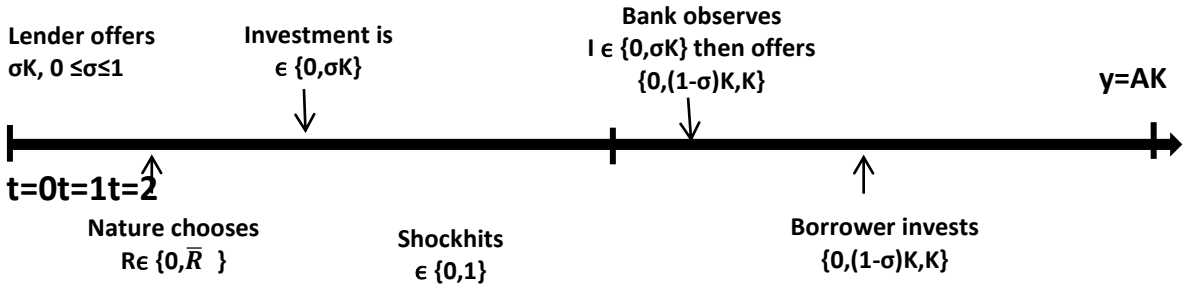
and the probabilities that the bank offers a second-period financing (beginning at $t=1$). Both the repayment schedule and the probabilities are conditional on first period investment. These probabilities are defined in the spirit of Bolton and Scharfstein (1990) and Stiglitz and Weiss (1983). Let $x_1 \in [0,1]$ be the probability that the foreign bank gives the domestic entrepreneur funds at date $t=1$ to continue the project (production stage) if the observed first period investment is positive (i.e. σK), and let $x_2 \in [0,1]$ be the probability that the foreign bank terminates the relationship if the observed first period investment is 0. Note that it is easy to show that $x_1 = 1$[xi]; thus, to ease the notation, we set $x_1 = 1$ and $x_2 = x$. The choice of these variables determines future borrowing and the ability and willingness to repay.

The model developed here is a self-selection model like those of Rothschild and Stiglitz (1976) or Stiglitz and Weiss (1981). However, our model has three additional complications. First, the information frictions are such that the entrepreneur cannot be forced to invest. Second, non-investing entrepreneurs cannot be separated from investing entrepreneurs in the selection process; hence, standard sorting devices, such as collateral (Bester (1985) and Bester (1987)), cannot be used to separate a bad entrepreneur from a good one. Third, I allow for arbitrary "high" probability of the entrepreneur being of a good type (i.e., ρ_0 high). Targeting such a good type by a foreign bank creates more moral hazard and leads to additional inefficiencies. These three complications lead to a perfect Bayesian equilibrium in which the bank does not continue lending to the borrower if first-period observed investment is sufficiently low (I=0).

3.2 Timing

The timing of events in this environment is represented by Figure 1. First, the bank decides whether to fund the project. If so, it provides $\sigma K \leq K$. Second, after the transfer is made, nature chooses $R \in \{0, \bar{R}\}$. Third, the domestic entrepreneur decides to behave or to misbehave. Then, the investment shock, ε, hits. Prior to its second-period decision, the bank observes the investment level, $I \in \{0, \sigma K\}$, updates its beliefs about R, then decides whether to continue funding or to terminate the relationship. If it decides to continue, it provides second-period funding $(1-\sigma)K$ or K; then the borrower decides whether to behave or not. If she behaves, output y is realized, and the borrower pays back the lender at the end of the second period.

Figure 1: Timing



4.Bayesian Strategies and Bayesian Equilibria

In this section, two potential equilibrium candidates, separating and pooling, are discussed, and athe conditions under which these equilibria might exist are nalyzed. In each case, given σ and Z, we propose an equilibrium loan contract (strategies for the bank and the entrepreneur as well as the outcome) and discuss the necessary conditions for that equilibrium to hold. In the absence of a perfect signal, the bank will only rely on its updated belief to make its second-period decision to stop or continue. At $t = 2$, the bank's beliefs that the borrower behaved given that $I = 0$ (i.e.,

$\rho_1 = Pr(R=0 \mid I=0)$) are given by

$$\rho_1 = \frac{\mu\rho_0}{1-\rho_0+\mu\rho_0}. \quad (4)$$

Note that $\rho_1 < \rho_0$

4.1Separating Equilibrium

At $t=0$, the lender offers a first installment, σK. If $I=0$ after one period, the lender does not offer a second installment. If $I=\sigma K>0$, the lender makes a second installment of $(1-\sigma)K$. The borrower accepts the contract. If $R=0$, the borrower invests the first installment in the project. If $\varepsilon=1$, she invests the second installment. If $R=\overline{R}$, she does not invest the first installment. Under what conditions is this an equilibrium? To answer this question, we solve for the equilibrium using backward induction.

In the second period (i.e., $t=1$), if $I=\sigma K$, lender's expected repayment is βZ. The bank will make the second installment $(1-\sigma)K$ if and only if

$$\beta Z \geq (1-\sigma)K. \quad (5)$$

This scenario effectively treats the first installment as sunk. The payoff to the (R=0)-type in this case will be $\beta[AK-Z]$ and she will continue as long as

$$AK \geq Z. \quad (6)$$

Condition (6) also represents the limited liability constraint.[xii]
If $I=0$, the bank's expected repayment is $\rho_1\beta Z_0$, and it will lend K if

$$\rho_1\beta Z_0 \geq K. \quad (7)$$

The expected payoff to ($R=0$)-type borrower is $\beta[AK-Z_0]$, and she will continue if and only if

$$AK \geq Z_0. \quad (8)$$

Condition (8) stands for the limited liability constraint, too. (If $R=\overline{R}$, the borrower misbehaves and gets a payoff $\overline{R}K$).

Lemma 1: For the equilibrium to be as described above, it must be the case that

$$\frac{1}{\beta\rho_1} = \frac{1-\rho_0+\mu\rho_0}{\beta\mu\rho_0} > A. \quad (9)$$

Lemma 1 implies that there is no repayment Z_0 that an $(R=0)$-type borrower could

make that the bank will find profitable. Under this condition, if $I = 0$, the bank stops lending.

In the first period (i.e., at $t = 0$), the expected repayment in this equilibrium is

$$\beta\rho_0(1-\mu)[\beta Z - (1-\sigma)K].$$

It follows that the bank will make the first installment if and only if

$$\beta\rho_0(1-\mu)[\beta Z - (1-\sigma)K] \geq \sigma K \tag{10}$$

or,

$$Z \geq \frac{1}{\beta}[\frac{\sigma}{\beta\rho_0(1-\mu)} + (1-\sigma)]K. \tag{11}$$

Note that if condition (11) holds condition (5) must hold.

The expected payoff to a ($R = 0$)-type borrower in this equilibrium is

$$\beta^2(1-\mu)[AK - Z],$$

and so she will enter the relationship as long as condition (6) holds. The expected payoff to an ($R = \overline{R}$)-type borrower in this equilibrium is $\overline{R}\sigma K$. For the equilibrium to be separating , the ($R = \overline{R}$)-type should not have an incentive to mimic the other type by investing the first installment just to get the second installment. A sufficient condition for no mimicking is

$$\overline{R}\sigma K \geq \beta(1-\mu)\overline{R}(1-\sigma)K$$

$$\sigma \geq \frac{\beta(1-\mu)}{1+\beta(1-\mu)} = \underline{\sigma}. \tag{12}$$

So, the first installment must be sufficiently large. Overall, the restrictions on σ and Z for this equilibrium to exist are given by conditions (6), (9), (11), and (12).

Combining conditions (6) and (11), for there to exist a Z that is consistent with this equilibrium, σ must satisfy

$$\sigma \leq \frac{\beta\rho_0(1-\mu)(\beta A-1)}{1-\beta\rho_0(1-\mu)} = \overline{\sigma}. \tag{13}$$

Conditions (12) and (13) impose upper and lower bounds on σ: if σ is too low (i.e.,

smaller than $\underline{\sigma}$), mimicking will occur, and no information will be derived from the initial loan. If σ is too high (i.e., bigger than $\bar{\sigma}$), the bank exposes her self to much risk, and no mutually beneficial contract exists.

Note finally that condition (9) implies

$$\beta A - 1 < \frac{1-\rho_0}{\rho_0 \mu}. \tag{14}$$

The following proposition summarizes the conditions under which a separating equilibrium exists.

PROPOSITION 1: *A separating equilibrium exists if and only if* $\beta A - 1 < (1-\rho_0)/\rho_0\mu$. *In this case, the lender offers a first installment,* σK, *at* $t = 0$ *where* $\underline{\sigma} \le \sigma \le \bar{\sigma}$. *If no investment is observed after one period the contracting relationship is terminated, which leads to a sudden stop. Otherwise the lender provides additional capital. The borrower accepts the contract and invests the first and second installments only if* $R = 0$.

4.2 Pooling Equilibrium

At $t = 0$ the bank offers the following contract: a first installment, σK, is provided in the first period. If $I = 0$ after one period, the bank does not offer a second installment. If $I = \sigma K > 0$, it makes a second installment of $(1-\sigma)K$. The borrower accepts the contract and invests the first installment in the project for any R. If $\varepsilon = 1$, she invests the second installment if and only if $R = 0$.

First, note that in this candidate for pooling equilibrium, there is no updating of beliefs. That is,

$$\rho_0 = \rho_1. \tag{15}$$

In the second period, if $I = \sigma K$, the bank's expected payoff is $\rho_0 \beta Z$, and it will offer the second installment if and only if

$$\rho_0 \beta Z \ge (1-\sigma)K. \tag{16}$$

If $R = 0$, the expected payoff to the borrower is $\beta[AK - Z]$, and she will continue if and only if condition (6) holds. If $R = \bar{R}$, the borrower gives up her period-one investment (i.e., the project is discontinued) and misbehaves (i.e., uses the additional funding she gets in the alternative investment opportunity). In this case, her expected payoff is $(1-\sigma)K\bar{R}$.

If $I = 0$, bank's expected repayment is $\rho_0 \beta Z_0$, and it will lend K to restart the project if

$$\rho_0 \beta Z_0 \ge K. \tag{17}$$

The borrower will continue in the second period if $R = 0$ and as long as condition (8) holds. If $R = \overline{R}$ and additional funds are provided in the second period, the borrower would divert them since (by assumption) $\overline{R} > A$ and her payoff would be $\overline{R}K > 0$. Note that this possibility is off the equilibrium path.

Lemma 2: For the equilibrium to be as described above, it must be the case that

$$\frac{1}{\beta\rho_0} > A. \tag{18}$$

That is, there is no repayment scheme Z_0 that an ($R = 0$)-type borrower could make that the bank would find profitable. Under this condition, if $I = 0$, the bank stops lending.

In the first period, the expected repayment in this equilibrium (i.e., where condition (18) holds) is

$$\beta(1-\mu)[\rho_0[\beta Z - (1-\sigma)K] - (1-\rho_0)[(1-\sigma)K]].$$

It follows that the bank will make the first installment if and only if

$$\beta(1-\mu)[\rho_0\beta Z - (1-\sigma)K] \geq \sigma K$$

or,

$$Z \geq \frac{1}{\rho_0\beta}[\frac{\sigma}{\beta(1-\mu)} + (1-\sigma)]K. \tag{19}$$

Note that if condition (19) holds condition (16) must hold.

The expected payoff to ($R=0$)-type borrower in this equilibrium is $\beta^2(1-\mu)[AK - Z]$, so she will enter the relationship as long as condition (6) holds. The expected payoff to ($R = \overline{R}$)-type borrower in this equilibrium is $\beta(1-\mu)\overline{R}(1-\sigma)K$.

For this to be a pooling equilibrium, the ($R = \overline{R}$)-type prefers to mimic the other type by investing the first installment just to get the second installment. A sufficient condition for mimicking is

$$\overline{R}\sigma K \leq \beta(1-\mu)\overline{R}(1-\sigma)K$$

or,

$$\sigma \leq \frac{\beta(1-\mu)}{1+\beta(1-\mu)}. \tag{20}$$

Overall, the restrictions on σ and Z for this equilibrium to exist are given by conditions (6), (18), (19), and (20). However, for there to exist a Z that is consistent with this equilibrium, σ (by combining conditions (6) and (19)) must satisfy

$$\sigma < \frac{\beta(1-\mu)(\beta\rho_0 A-1)}{1-\beta(1-\mu)}. \tag{21}$$

Note, however, that condition (18) implies that

$$\beta\rho_0 A < 1. \tag{22}$$

So, condition (21) cannot hold for $\sigma > 0$, implying that this pooling equilibrium cannot exist. The following proposition summarizes these findings.

PROPOSITION 2: *There is no pooling equilibrium such that the ($R = \overline{R}$)-type borrower mimics the ($R = 0$)-type in the first period.*

5.Optimal Contract

In the previous section the conditions necessary for the existence of a separating equilibrium contract were derived. These conditions include lower and upper bounds on σ, a lower bound on Z and condition to rule out any payment scheme involving Z_0. In this section, we narrow our analysis and investigate the most efficient combination of all these variables. In other words, the focus is on finding the optimal contract that maximizes the bank's expected payoff, subject to the borrower's incentive and participation constraints.

5.1 Equilibrium Contract

The optimal contract maximizes the expected payoff of the foreign bank, subject to the following constraints: (i) incentive compatibility (hereafter IC): the ($R = 0$)-type invests at dates 0 and 1, and ($R = \overline{R}$)-type misbehaves at dates 0 and 1; (ii) limited liability: the contract does not violate limited liability; (iii) individual rationality: the entrepreneur opts to sign the contract at date 0. Formally, the problem is the following:

$$\max_{\{\sigma,Z,Z_0,x\}} \beta\{\rho_0((1-\mu)[\beta Z-(1-\sigma)K]+x\mu[\beta\rho_1 Z_0-K])-(1-\rho_0)(\sigma K+xK)\}-\sigma K$$

s.t (6), (8), and

$$\sigma K\overline{R} \geq \beta\{(1-\mu)(1-\sigma)K\overline{R}+\mu x K\overline{R}\} \tag{23}$$

$$\beta\{\rho_0\beta((1-\mu)[AK-Z]+x\mu[AK-Z_0])+(1-\rho_0)(\sigma K\overline{R}+x\beta K\overline{R})\} \geq 0 \tag{24}$$

where x is the the probability that the bank terminates the relationship if the observed first-period investment is 0.

To make the contract incentive compatible, the lender designs the contract so that the ($R = \overline{R}$)-type does not mimic the (R=0)-type. The incentive constraint (23) ensures that when $R = \overline{R}$ the domestic entrepreneur (bad type) does not mimic the good type in the

first period.

Note that the limited-liability constraints (6) and (8) imply that the individual-rationality constraint (24) is not binding. The following lemma simplifies the analysis of the optimal contract. See appendix B for the way in which this lemma simplifies the maximization problem.

Lemma 3: *The incentive compatibility constraint (23) is binding at an optimum.*

Lemma 3 is a typical feature of contracting problems. Using Lemma 3 and solving the *binding* IC constraint (23) for σ, we get

$$\sigma = \frac{\beta[1-(1-x)\mu]}{1+\beta(1-\mu)}. \tag{25}$$

By substituting (25) into the bank's objective function, the maximization problem simplifies to

$$\max_{Z,Z_0,x} \{\beta[\rho_0(1-\mu)[\beta Z - \frac{1}{1+\beta(1-\mu)}K] - (1-\rho_0)[\frac{\beta(1-\mu)}{1+\beta(1-\mu)}]K]$$

$$-[\frac{\beta(1-\mu)}{1+\beta(1-\mu)}K + h(K, Z_0, \beta, \mu, \rho_0)x]\}$$

subject to conditions (6) and (8)
where

$$h(K, Z_0, \beta, \mu, \rho_0) = \beta\{\frac{\mu(1-\rho_0(1-\mu)\beta)+(1-\rho_0)(1+\beta)}{1+\beta(1-\mu)}K - \rho_0\mu[\beta\rho_1 Z_0 - K]\}.$$

Note that $h(K, Z_0, \beta, \mu, \rho_0) \geq 0$ if and only if

$$Z_0 \leq f(\rho, \mu)K \tag{26}$$

where

$$f(\rho, \mu) = \frac{\mu[1-\rho_0\beta(1-\mu)]+(1-\rho_0)(1+\beta)+\rho_0\mu[1+\beta(1-\mu)]}{\rho_0\mu\beta[1+\beta(1-\mu)]\rho_1}. \tag{27}$$

Recall that by the limited liability constraint (8), we have $AK \geq Z_0$. So, we need to compare A and $f(\rho, \mu)$, and, if it turns out that $f(\rho, \mu) \geq A$, then condition (26) is satisfied (because $Z_0 \leq AK \leq f(\rho, \mu)K$). In appendix B, we show that $f(\rho, \mu) \geq A$. Consequently, condition (26) is satisfied, which implies that $h(Z_0, K, \beta, \mu, \rho_0) \geq 0$. It follows immediately from the maximization problem that $x = 0$, and the objective function simplifies to

$$\max_{Z}\{\beta[\rho_0(1-\mu)[\beta Z-\frac{1}{1+\beta(1-\mu)}K]-(1-\rho_0)[\frac{\beta(1-\mu)}{1+\beta(1-\mu)}]K]$$
$$-\frac{\beta(1-\mu)}{1+\beta(1-\mu)}K\}$$

Since the bank's payoff is linear in Z, it follows that $Z^* = AK = y$.
Note that using Lemma 3 together with the result $x=0$, and solving the IC, (23), constraint for σ, we get

$$\sigma^* = \frac{\beta(1-\mu)}{1+\beta(1-\mu)}. \tag{28}$$

It is important to mention that σ^* satisfies both condition (12) and (13). More interestingly, $\sigma^* = \underline{\sigma}$, which means that the lender provides the minimum amount of capital necessary in the first period to induce separation.

In sum, the optimal contract can be represented as: $\{x_1, x_2, \sigma, Z\} = \{1,0,\sigma^*, y\}$. That is, in the first period, the foreign bank provides a fraction of capital equal to σ^* which is smaller than half. After observing the first period's outcome, the bank will provide additional funding if and only if investment is positive, i.e. $x_1 = 1$ and $x_2 = 0$. At the end of period 2, the entrepreneur pays back the loan, and this repayment is y.

Finally, I must determine the conditions under which the bank earns non-negative profit. Given the optimal contract, the bank's expected profits are

$$\beta[\rho_0(1-\mu)[\beta AK-\frac{1}{1+\beta(1-\mu)}K]-(1-\rho_0)[\frac{\beta(1-\mu)}{1+\beta(1-\mu)}]K]-\frac{\beta(1-\mu)}{1+\beta(1-\mu)}K. \tag{29}$$

Thus, for the bank to finance the project at date 0, the prior, ρ_0, should satisfy

$$\rho_0 \geq \frac{1+\beta}{\beta(1+A[1+\beta(1-\mu)])-1} = \underline{\rho}_0. \tag{30}$$

As a result, some positive net present value projects may not be funded.

I summarize these results in the following proposition.

PROPOSITION 3: *The foreign bank lends at date $t=0$, if and only if $\rho_0 \geq \underline{\rho}_0$. In this case, it offers $\sigma^* = \frac{\beta(1-\mu)}{1+\beta(1-\mu)}$ in the first period; sets the probabilities of additional funding in the second period to $x_1 = 1$ if $I > 0$, and $x_2 = 0$ if $I = 0$; and requires a repayment of $Z^* = y$.*

5.2Comparative Statics

Having characterized the equilibrium contracts in the presence of information frictions, I now perform two types of comparative statics exercises, with respect to the breakpoint between short- and long-term loans, σ^*, and the bank's participation condition defined by (30) (i.e. nonnegative profit). The proofs are simple consequences of the characterization of the the optimal contract (differential of σ^* and $\underline{\rho}$ with respect to the parameters of the model) and are therefore omitted.

5.2.1 Debt Maturity

***Changes in the discount rate*.**

Corollary 1. *The equilibrium share of long-term loan is an increasing and concave function of the discount rate. That is*,

$$\frac{\partial \sigma^*}{\partial \beta} = \frac{1-\mu}{[1+\beta(1-\mu)]^2} > 0 \quad and \quad \frac{\partial^2 \sigma^*}{\partial \beta^2} = \frac{-2(1-\mu)}{[1+\beta(1-\mu)]^3} < 0. \tag{31}$$

A decrease in the short-term interest rate (i.e., an increase of β) motivates more long-term lending (i.e., an increase of σ^*) since short-term lending is less attractive to the bank. This implies that the bank will be better off if it finances the project with a long-term loan only. However, because of the information frictions and the risk associated with them, the bank cannot finance the project by long-term debt alone. The concavity property emphasizes this fact. This result is further confirmed by the fact that, as we saw in condition (28), $\sigma^* < 0.5$ for all $\mu \in [0,1]$; that is, the long-term debt is smaller than the short-term debt. This finding matches the stylized fact that the share of short-term debt in the overall debt is higher in emerging markets.

Changes in the distribution of the aggregate uncertainty.

Corollary 2. *The equilibrium share of long-term loan is a decreasing and concave function of the aggregate uncertainty. That is*,

$$\frac{\partial \sigma^*}{\partial \mu} = \frac{-\beta}{[1+\beta(1-\mu)]^2} < 0 \quad and \quad \frac{\partial^2 \sigma^*}{\partial \mu^2} = \frac{-2\beta^2}{[1+\beta(1-\mu)]^3} < 0. \tag{32}$$

This result implies that the increasing likelihood of bad shock reduces the share of long-term lending in the overall debt; That is, decreases σ^*. It is important to mention here that, because the optimal contract induced separation, there is no risk of mimicking, and the bank is not taking an additional risk by lowering the long-term lending. Condition (32) also implies that, although the high likelihood of a negative shock suggests short-term lending is safer, the lender will consider long-term lending, too. This is because the foreign bank is still interested in learning the type of the borrower.

5.2.2 Foreign Bank's Participation

The foreign bank will finance the project if and only if condition (30) is satisfied. This implies that, although the prior does not affect the allocation of the loan between short- and long-term (i.e., σ independent of ρ_0), it is a key determinant of the optimal contract. So, it is very important to address the sensitivity of condition (30) to the model's parameters.

Changes in the discount rate.

Corollary 3. *The foreign bank's participation condition is decreasing and convex in the discount rate. That is*,[xiii]

$$\frac{\partial \underline{\rho}_0}{\partial \beta} = \frac{-[(2+A)+\beta A(1-\mu)(2+\beta)]}{[\beta(1+A[1+\beta(1-\mu)])-1]^2} < 0 \quad and \quad \frac{\partial^2 \underline{\rho}_0}{\partial \beta^2} > 0. \tag{33}$$

The reasoning behind this result is that as the interest rate decreases (i.e., β increases), loans are more affordable. And, because the optimal contract induces separation, only good types of borrowers are attracted. The lower bound on the prior reuired by the bank, $\underline{\rho}$, decreases, and consequently, more projects with positive net present value are funded. It is important to note here that, while the change in the discount rate decreases the prior, it increases (as discussed in Corollary 1) the long-term lending ($\uparrow \sigma$).

***Changes in the distribution of aggregate uncertainty*.**

Corollary 4. *The foreign bank's participation condition is increasing and convex in the distribution of the aggregate uncertainty. That is*,

$$\frac{\partial \underline{\rho}}{\partial \mu} = \frac{\beta^2 A(1+\beta)}{[\beta(1+A[1+\beta(1-\mu)])-1]^2} > 0 \quad and \quad \frac{\partial^2 \underline{\rho}}{\partial \mu^2} = \frac{2\beta^2 A(1+\beta)\beta^2 A}{[\beta(1+A[1+\beta(1-\mu)])-1]^3} > 0. \tag{35}$$

As the aggregate uncertainty increases, the willingness of the bank to finance the project decreases.

6.Sudden Stop

Proposition 3 implies that there is an ex-post inefficiency; the project is discontinued when first-period observed investment is 0, even if $\beta A > K$ and it is profitable to operate. This scenario implies that, in this model, the size or the overall debt burden is an important determinant of the financial distress a country might experience. In fact, the model shows that when first period investment is zero, there is no repayment scheme Z_0 that makes the continuation of the relationship attractive to the bank. This result is consistent with previous empirical work. Rodrik and Velasco (1999), for example, find

that crisis probabilities are increasing in the overall debt burden (measured by the debt-GDP ratio). This ex-post inefficiency implies also that, because of the information frictions, a good borrower is not able distinguish herself from a bad borrower, leading to a termination of the contracting relationship.

6.1 Sudden Stop and Debt Maturity

As discussed earlier, σ in this model represents the long-term debt, and $(1-\sigma)$ represents short-term debt. So, a decrease in σ implies an increase of the short-term (and a decrease of long-term) debt. The existing literature on financial crises suggests that short-term borrowing was a causal factor in the episodes of sudden stops in emerging market economies. Rodrik and Velasco (1999), for example, find that longer term borrowing is associated with a lower probability of crisis. One interpretation of Rodrik and Velasco's result is that the medium- and long-term debt stock is correlated with omitted country attributes that increase creditworthiness and reduce the risk of crises. The results of the present study show that, even with long term debt (i.e., $\sigma > 0$), emerging market economies are vulnerable to crises. This finding is supported by Calvo (1998)'s model, which shows that capital-market crises could take place even if most capital inflows take the form of foreign direct investment.

It is important to note that the external debt structure derived in the present framework, characterized by information frictions, might not be different from the debt structure of an economy characterized by deeper financial markets. In fact, as economies get richer and financial markets become deeper (through financial liberalization, lower monitoring costs, good information environment, etc.), the external debt structure gets tilted towards short-term liabilities, a view widely supported in the literature. One reason for this shift is that countries that go on a borrowing binge are forced to shorten the maturity of their external liabilities. Our model shows that, with limited information, the debt structure is very similar as the information asymmetries tilt external debt towards a smaller share of long-term loans. This result implies that short-term debt is not necessarily a feature of financial markets with lack of information. The fact that σ^* does not depend on ρ_0 supports this view.

6.2 What is International about All That?

The above discussion could have been carried out in terms of a contracting relationship between any two agents in a country. And the same narrative would apply with only minor presentation changes. However, although the threat of contractual termination occurs in a national context, there are important international characteristics that would be absent. As a general rule,

1. domestic lenders, as opposed to international lenders, are most likely able to learn about the exogenous shocks that may hit the investment and then infer easily the alternative investment opportunities available to the borrower. In particular, domestic can verify the state of the world at lower cost than international lenders. Thus, in a way, domestic lenders are more able to achieve first-best contracts.

2. For legal and institutional reasons, capital mobility across countries is markedly more responsive to cyclical fluctuations than the mobility within a country; thus, expected international transfers (loans and/or payments) are more reflective of more international capital market conditions than local capital market conditions.

Point 1 implies that assessing non-natural risks could be more costly in crisis-stricken economies because the financial and political institutions in many of them (e.g., Mexico, Korea, Indonesia) are in a state of flux and, in some cases (e.g., Argentina), because they have a relatively young economic system. As shown in Calvo and Mendoza (1997), high risk-assessment costs make herding more likely. This means that, if an international lender withdraws from the country, it will be very difficult for the domestic entrepreneur to find another to replace it.

Point 2 implies that a country hit by a shock is less likely to avoid the crisis. Higher international capital mobility together with the high reliance on foreign capital implies that, during crisis times, withdrawals of international lenders are instantaneous and more harmful than termination by domestic lenders.

The above discussion leads to the conjecture that a cut in foreign loans contributes to deeper and longer-lasting financial crises than a cut in domestic loans. Furthermore, because of informational limitations, foreign lenders cannot distinguish among borrowers from the same country and treat them all as equally risky. In fact, the policy of sovereign ceilings followed by rating agencies, in which no single company can have a rating higher than the government of its country, suggests that this is indeed the case.

7.Conclusion

The debate on the causes of sudden stops will undoubtedly go on for a long time. Bad luck, in the form of exogenous shocks and from nature, and bad policy, in the shape of poor regulation and imprudent macro policies, doubtless carry some of the blame. But these are not the sole causes of sudden stops. The main message of this paper is that information frictions were at the center of the financial crises of the 1990s and that, even though short-term debt is a crucial ingredient of financial fragility, it is not the key factor in sudden stops.

In the aftermath of the crises of the 1990s, the reaction, particularly from multilateral lenders, has been to call for more prudent monetary and fiscal policies and greater supervision and transparency in local financial markets. But there is limited agreement on what macroeconomic policies are “appropriate” in this context. Analysts of the Asian episodes, for instance, seem to be evenly divided between those who think that liberalization of capital markets should go on and those who claim that some restrictions must be imposed on international capital flows. The current emphasis on strengthening domestic financial systems also glosses over the practical difficulties. Putting in place an adequate set of prudential and regulatory controls to prevent moral hazard and excessive risk-taking in the banking system is much more easily said than done. Even the most advanced countries fall considerably short of the ideal.[xiv]

The model developed in this paper matches many of the qualitative properties of sudden stops that have been documented. Moral hazard seems to be a good foundation for the analysis of financial crisis. One key policy goal to deal with information frictions, like those in this paper, is to improve information channels. Recent efforts by the IMF and developing countries to improve data dissemination move in the direction of this goal.[xv]

Endnotes

[i]According to Calvo (1998), the expression sudden stop was inspired by a bankers adage that it is not speed that kills, it is the sudden stop.

[ii]See for example, Arellano and Mendoza, 2002.

[iii]For an extensive survey of these models see the November, 1996 and June, 2000 symposia issues of the *Journal of International Economics*, or the NBER volumes edited by Edwards (2000), Krugman (2000) and Frankel and Edwards (2002).

[iv]See, for instance, Sachs Tornell, and Velasco (1996) for Mexico, Chang and Velasco (1998) and Radelet and Sachs (1998) for East Asia. Others, such as Corsetti, Pesenti and Roubini(1999) have emphasized fundamental economic imbalances as the main source of fragility in East Asia.

[v]See also, Cole and Kehoe (1996), Furman and Stiglitz (1998), Obstfeld (1998), Eichengreen and Hausmann (1999), and Feldstein (1999).

[vi]Hong Kong, Indonesia, Malaysia, Philippines, Taiwan, Thailand, and Korea.

[vii]Whereas domestic loans are generally supported by substantial collateral, the assets that can be appropriated in the event of a sovereign's default are generally negligible. For this reason one must look beyond collateral to find incentives for repayment.

[viii]Note that first period capital input might be zero or K. If it is zero, then the second period capital input must be equal to K for the project to be productive.

[ix]ε could take various forms: investment-specific technology shock in the intermediate good sector, productivity shock, non-concluding R&D activities, weather-induced shocks, etc. Or, it could just be a ``mistake" by workers.

[x]This result may not be inefficient. The social surplus is the sum of the lender's and the borrower's payoffs. If $R = \overline{R}$, and since $\overline{R} > A$, then it is socially efficient to divert the second period funds.

[xi]If $I > 0$, it must be that the borrower is $(R = 0)$ type with return being positive, $y > 0$, at $t = 2$.

[xii]Note that ($R = \overline{R}$)-type's payoff in period 2 is $\overline{R}(1-\sigma)K$, and she will accept the second-period funding with no constraint since she will not pay at $t = 2$ anyway. Note that this possibility is off the equilibrium path.

[xiii]See appendix B for the expression of $\dfrac{\partial^2 \underline{\rho}_0}{\partial \beta^2}$

[xiv]Indeed, banking crises have taken place in countries such as Sweden and Japan in the 1990s, and in *all* developed economies in 2007-08.

[xv]The IMF's work on data dissemination standards began in October 1995, when the Interim Committee (now the International Monetary and Financial Committee or IMFC) endorsed the establishment by the Fund of standards to guide members in the dissemination to the public of their economic and financial data. Those standards were to consist of two tiers: the General Data Dissemination System (GDDS), which would apply to all Fund members, and the Special Data Dissemination Standard (SDDS), for those member countries having or seeking access to international capital markets. The SDDS was approved by the IMF Executive Board in March 1996 and the GDDS in December 1997.

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Appendix

A. Lender's maximization Problem

To see how Lemma 3 simplifies the maximization problem, note that by Lemma 3, we solve the IC constraint for σ to get

$$\sigma = \frac{\beta[1-(1-x)\mu]}{1+\beta(1-\mu)} \tag{36}$$

Next, plug (36) into the lender's expected payoff, EU , (for easy notation I ignore the max operator)

$$EU = \beta\{\rho((1-\mu)[\beta Z - (1-\frac{\beta[1-(1-x)]\mu}{1+\beta(1-\mu)})K] + x\mu[\beta b Z_0 - K])$$
$$-(1-\rho)(\frac{\beta[1-(1-x)\mu]}{1+\beta(1-\mu)} + x)K\}$$

$$= \beta\{\rho[(1-\mu)[\beta Z - (\frac{1-\beta x\mu}{1+\beta(1-\mu)})K] + x\mu[\beta b Z_0 - K]] - (1-\rho)[\frac{x(1+\beta)+\beta(1-\mu)}{1+\beta(1-\mu)}]K\}$$

$$= \beta\{\rho(1-\mu)[\beta Z - \frac{1}{1+\beta(1-\mu)}K] + \frac{\rho(1-\mu)\beta\mu x}{1+\beta(1-\mu)}K + \rho\mu\beta b Z_0 x$$

$$-\rho x\mu K - \frac{(1-\rho)(1+\beta)x}{1+\beta(1-\mu)}K - \frac{(1-\rho)\beta(1-\mu)}{1+\beta(1-\mu)}K\}$$

let $g(x)$ a function that regroups all terms (in the above function) involving x ; thus,

$$g(x) = [\frac{\rho(1-\mu)\beta\mu}{1+\beta(1-\mu)}K - \frac{(1-\rho)(1+\beta)}{1+\beta(1-\mu)}K + \rho\mu\beta b Z_0 - \rho\mu K]x$$

The first and second terms in the square brackets on the right hand side can be arranged to get

$$-\frac{[1+\beta-\rho(1+\beta+(1-\mu)\mu\beta)]}{1+\beta(1-\mu)}K$$

It follows that

$$g(x) = [\rho\mu(\beta b Z_0 - K) - \frac{[1+\beta-\rho(1+\beta+(1-\mu)\mu\beta)]}{1+\beta(1-\mu)}K]x$$
$$= \frac{[\rho\mu\beta b Z_0 - \rho\mu K][1+\beta(1-\mu)] - (1+\beta)K + \rho[1+\beta+(1-\mu)\mu\beta]K}{1+\beta(1-\mu)}x$$
$$= \frac{\rho\mu\beta b Z_0[1+\beta(1-\mu)] - [\rho\mu[1+\beta(1-\mu)] + (1+\beta) - \rho[1+\beta+(1-\mu)\mu\beta]]K}{1+\beta(1-\mu)}x$$
$$= -h(Z_0, K)x$$

where $h(Z_0, K) = \dfrac{[\rho\mu[1+\beta(1-\mu)]+(1+\beta)-\rho[1+\beta+(1-\mu)\mu\beta]]K - \rho\mu\beta bZ_0[1+\beta(1-\mu)]}{1+\beta(1-\mu)}$

.

Using the result $x = 0$ and plug it into (36) we get (28).

B. Proof of condition (26)

$$f(\rho,\mu) = \frac{\mu[1-\rho_0\beta(1-\mu)]+(1-\rho_0)(1+\beta)+\rho_0\mu[1+\beta(1-\mu)]}{\rho_0\mu\beta[1+\beta(1-\mu)]\rho_1}$$
$$= \underbrace{\frac{\mu[1-\rho_0\beta(1-\mu)]+(1-\rho_0)(1+\beta)+\rho_0\mu[1+\beta(1-\mu)]}{\rho_0\mu[1+\beta(1-\mu)]}}_{m(\rho,\mu)} \times \frac{1}{\beta\rho_1} \qquad (37)$$

By (9), we have that $\dfrac{1}{\beta\rho_1} > A$. We also have that $A > 1$. So, to complete our proof, we need to show that $m(\rho,\mu) \geq 1$.

$$m(\rho,\mu) \geq 1$$
$$\mu + (1-\rho_0)(1+\beta) + \rho_0\mu \geq \rho_0\mu[1+\beta(1-\mu)]$$
$$\mu + (1+\beta) - \rho_0(1+\beta) + \rho_0\mu \geq \rho_0\mu + \rho_0\mu\beta(1-\mu)]$$
$$\mu + (1+\beta)(1-\rho_0) \geq \rho_0\mu\beta(1-\mu) \qquad (38)$$

We have that $\mu > \mu\rho_0\beta(1-\mu)$. So, $m(\rho,\mu) \geq 1$ and (26) is satisfied.

C. Corollary 3

$$\frac{\partial^2 \underline{\rho}_0}{\partial\beta^2} = \frac{4(1+\beta)+2\beta A(1-\mu)(1+\beta)^2+2A(1+\beta)[1-\beta(1-\mu)]}{[\beta(1+A[1+\beta(1-\mu)])-1]^3}$$
$$+\frac{2A(1-\mu)[2\beta(2+A)+\beta^2A(1-\mu)(3+\beta)-(1+\beta)]}{[\beta(1+A[1+\beta(1-\mu)])-1]^3} > 0 \qquad (39)$$